Technical Comments

Comment on "Performance of Quasi-Steady MPD Thrusters at High Powers"

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IN a recent Synoptic[1] and full paper[2] Malliaris et al. have shown that quasi-steady MPD arc performance appears to be limited by a critical value of $(J^2/\dot{m})$ such that $(J^2/\dot{m})_c = 1/b[2eN_0V_i/M]^{1/2}$; where all notation is identical to that of Ref. 1. They note that operation beyond the critical point becomes objectionable due to instabilities, sharp rise of voltage, erosion, and participation of spurious propellant. Noting that $M/N_0 = m_i$, where m_i is the mass of an ion, and $J^2/\dot{m} = v_e/b$, where v_e is the exhaust velocity, it is seen that the critical condition requires $(v_e)_c = [2eV_i/m_i]^{1/2}$ or that the exhaust velocity at the critical condition is given by the Alfvén critical velocity[3,4] (this result being equivalent to Eq. (7) of Ref. 2). A comparison of the computed Alfvén velocity and the measured exhaust velocity corresponding to $(I_{sp})_c$ of Table 2 of Ref. 1 is shown in Table 1. Since the analytic expression for $(J^2/\dot{m})_c$ is obtained by assuming a minimum power input which results in the equipartition of energy between ionization and kinetic energies,[2,4] a consequence of this model is the limiting of the exhaust velocity to the Alfvén velocity and, for a highly ionized exhaust stream, the limiting of the thrust efficiency to 50% or less. Thus from the analytic and experimental results presented in Ref. 1 it appears that the Alfvén critical velocity may have significance with regard to the performance of self-field quasi-steady arcs.

It is interesting to note that a similar critical condition accompanied by a sudden jump in voltage has also been observed in a steady applied field lithium-fueled MPD arc.[5,6] In this device, however, ion velocities more than twice as high as the Alfvén velocity have been measured at operating conditions below critical; i.e., operation at values of arc current, applied magnetic field, and input feed rate such that sudden voltage jumps are not observed. Both Doppler shift and energy analyzer techniques were used to measure ion velocities directly. The measured values agreed to within 15% with velocities deduced from thrust measurements,[6] indicating they were representative of the effective exhaust velocity. The thrust efficiency, for the fully ionized beam, varied from 25% to 45%.

It is not the purpose of this comment to define a performance limit for the quasi-steady arc, nor to compare quasi-steady and steady applied field arc performance. Its purpose is to point out that questions regarding the Alfvén critical velocity, limiting velocities, and limiting efficiencies which have been raised for applied field arcs[4] also appear relevant for self-field quasi-steady arcs.

Table 1 Comparison of Alfvén velocity and measured exhaust velocity at the critical condition

Entry no.	Propellant	$\dot{m}$, g/sec	$(v_e) \times 10^{-4}$, m/sec	$v_e = (2eV_i/m_i)^{1/2} \times 10^{-4}$, m/sec
1	Helium	0.7	2.9	3.3
2		1.5	2.5	3.3
3		4.1	2.7	3.3
4	Neon	1.6	1.2	1.4
5		4.0	1.2	1.4
6		8.5	1.3	1.4
7	Argon	1.1	0.90	0.87
8		2.2	0.92	0.87
9		5.6	0.90	0.87
10		12.0	0.88	0.87
11	Krypton	3.3	0.60	0.57
12		8.6	0.59	0.57
13		18.0	0.62	0.57
14	Xenon	4.1	0.49	0.42
15		10.4	0.49	0.42
16		22.3	0.47	0.42

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Comment on "Wind-Tunnel Magnus Testing of a Canted Fin or Self-Rotating Configuration"

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Nomenclature

A = true angle of attack
C_m = pitching moment coefficient
C_N = normal-force coefficient
C_n = yawing moment coefficient
C_{N_p} = Magnus-force derivative, $\partial C_n/\partial(pd/V)$
C_Y = side-force coefficient
d = reference length
$\{\bar{i}, \bar{j}, \bar{k}\}$ = unit vectors along $\{X, Y, Z\}$ axes
l_F = distance between Magnus force c.p. and c.g.
$\bar{l}$ = unit vector along velocity vector

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p = spin rate
V = freestream velocity vector
X, Y, Z = wind-tunnel balance axes
α = indicated angle of attack
β = angle of side slip
ε = angle between angle-of-attack plane and vertical plane

Subscript
o = indicates flow asymmetries

In a recent paper A. S. Platou of the Ballistic Research Laboratories offered a method for correcting Magnus measurements for normal force interaction.[1] Since the data used for the demonstration of his technique was drawn entirely from measurements that I had made, I feel that some additional comments are in order.[2]

While I do not disagree with Platou's statement that normal force interactions might exist in the preliminary data presented in Ref. 2, I felt that such interaction is manifested in a way other than that cited by Platou. The development of an algorithm for Magnus data correction may not be possible from the data contained in Ref. 2 alone.

Platou states in Ref. 1, "The existence of a moment at zero force is indicative of a couple and is due to the normal-force interaction term ($N \sin\varepsilon$) acting opposite to the fin Magnus force." The presence of a pure couple is by no means indicative of normal-force interaction. For example, one model of the Magnus effect on a body with canted fins actually predicts a pure couple.[3] Thus, to point to the presence of a pure couple as incontestible evidence of normal-force interaction seems rather arbitrary.

In Eqs. (4), (5), and (6) of Ref. 1, Platou provides interaction correction equations. Typical is Eq. (4) which is written below as

$$C_Y = C_N \sin\varepsilon + C_{N_p}(pd/V)\cos\varepsilon \tag{1}$$

where

$$\sin\varepsilon = (Cn - l_F C_Y)/lC_N \tag{2}$$

where ε is the angle between the plane defined by the balance axis and the flow vector and the plane defined by the balance axis and direction of the balance at nominal zero angle of attack. The term l is the distance obtained by dividing the pure couple by the normal-force interaction, $N \sin\varepsilon$; the term l_F is the location of the Magnus center of pressure from the center of gravity. Platou has rather arbitrarily set l_F as the distance from the center of gravity to the midchord of the stabilizer. In addition, it is not clear if Eqs. (1) and (2) are valid when both forebody and stabilizer are rotating as a unit. As Platou states, "In the case of a finned projectile, where the body rotates, l_F cannot be used."

An alternate approach will now be given for correcting the Magnus loads for normal-force interactions. In this method the interaction is evident not in the presence of a couple, but rather in the existence of residual yawing and pitching moments at zero angle of attack. Such moments indicate that there is a flow angularity, represented by $\{\alpha_0, \beta_0\}$ in Fig. 1.

Figure 1 indicates that the balance rotates through the nominal angle of attack, α about the Y axis. Because of the presence of flow angularity the model experiences an angle of attack A. The wind-tunnel balance resolves the aerodynamic force and moment into components along and about the Y axis, i.e., C_Y and C_n, and along and about the Z axis, i.e., $-C_N$ and C_n. However, the loads are interpreted (or defined) as being resolved in the angle-of-attack plane (along and about the vector $\bar{i}x$ $(\bar{l}\times\bar{i})$ and normal to the angle-of-attack plane (along and about the vector $(\bar{l}\times\bar{i})$. Since β_0 is not necessarily zero, the angle-of-attack plane (defined by $\{\bar{i}, \bar{l}\}$ is not necessarily coincident with the plane through which the balance rotates (defined by $\{\bar{i}, \bar{i}_0\}$). Nonzero $\{\alpha_0, \beta_0\}$ result in the true angle of attack, A, and the nominal angle-of-attack α being unequal. The use of the word "corrective" does not imply that the measurements available from the wind-tunnel balance are in error, but rather that these measurements must be redefined from the plane-of-balance rotation to the true angle-of-attack plane. Thus, the goal of the corrective equations

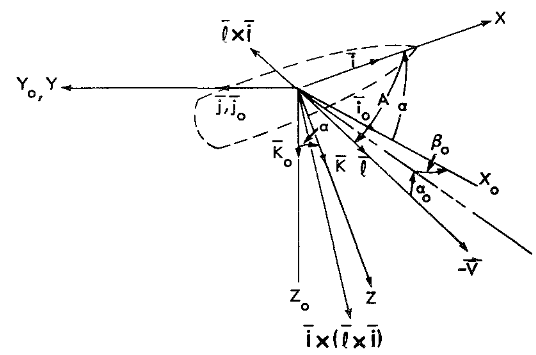

Fig. 1 Flow angularity geometry.

is to take balance-measured quantities $\{C_Y, C_n, \alpha\}$ and to "correct" or redefine these quantities with respect to the angle-of-attack plane. In doing so it will be found that there is a normal-force and pitching-moment interaction on side-force and yawing-moment measurements.

Such corrective equations have been developed by resolving the measured force and moment vectors, $\bar{C}_F$ and $\bar{C}_M$ from the balance axes,

$$\bar{C}_F = C_Y\bar{j} - C_N\bar{k} \tag{3a}$$

$$\bar{C}_M = C_m\bar{j} + C_n\bar{k} \tag{3b}$$

to the aerodynamic axes

$$\bar{C}_F = C_Y{}^c\left(\frac{\bar{l}\times\bar{i}}{|\bar{l}\times\bar{i}|}\right) - C_N{}^c\left(\frac{\bar{i}\times(\bar{l}\times\bar{i})}{|\bar{i}\times(\bar{l}\times\bar{i})|}\right) \tag{4a}$$

$$\bar{C}_M = C_m{}^c\left(\frac{\bar{l}\times\bar{i}}{|\bar{l}\times\bar{i}|}\right) + C_n{}^c\left(\frac{\bar{i}\times(\bar{l}\times\bar{i})}{|\bar{i}\times(\bar{l}\times\bar{i})|}\right) \tag{4b}$$

After carrying out the operations implied by Eqs. (3) and (4) the corrected side-force and yawing-moment coefficients become,

$$C_Y{}^c = (C_Y h + C_n k)/a \tag{5a}$$

$$C_n{}^c = (C_n h + C_m k)/a \tag{5b}$$

where

$$h = \cos\alpha_0 \cos\beta_0 \sin\alpha + \sin\alpha_0 \cos\alpha \tag{6a}$$

$$k = \cos\alpha_0 \sin\beta_0 \tag{6b}$$

$$a = (h^2 + k^2)^{1/2} \tag{6c}$$

and where the corrected coefficients are presented as functions of the corrected angle of attack, A, as

$$A = \tan^{-1}\left\{\frac{[(\cos\alpha_0 \cos\beta_0 \sin\alpha + \sin\alpha_0 \cos\alpha)^2 + (\cos\alpha_0 \sin\beta_0)^2]}{[\cos\alpha_0 \cos\beta_0 \cos\alpha - \sin\alpha_0 \sin\alpha]}\right\}^{1/2} \tag{6d}$$

The flow angularity variables $\{\alpha_0, \beta_0\}$ are obtained from the measured residual moments $\{C_{m_0}, C_{n_0}\}$ at a nominal angle of attack of zero:

$$\alpha_0 = C_{m_0}/C_{m_\alpha} \qquad \beta_0 = -C_{n_0}/C_{m_\alpha} \tag{7}$$

A possible advantage of this method over that presented in Ref. 1 is that the measured quantities, $\{C_Y, C_n, \alpha\}$ are transferred into the true Magnus quantities $\{C_Y{}^c, C_n{}^c, A\}$ without calculating or guessing at c. p. locations; further, it is applicable to the case of stabilizer spinning alone or body and stabilizer spinning as a unit.

There are two other minor disagreements that I have with the presentation in Ref. 1. In Fig. 1 of Ref. 1 the vector depicting the direction of the "negative side-force measuring direction" should be perpendicular to the balance axis and not to the flow direction. Under this circumstance Eq. (2), Ref. 1, becomes

$$\sin\varepsilon = \sin\beta/\sin\alpha \tag{8}$$

I also disagree with Platou's conclusion that a tare measurement be used to remove the interaction by "...subtracting zero-spin data at each angle of attack" although, he points out earlier, "...zero-spin data are difficult or impossible to obtain on self-rotating configuration...." In actual fact, zero-spin data are quite easy to obtain and were part of the program of which portions were reproduced in Ref. 2: The model is locked to the balance by means of a set screw. The principal objection to using zero-spin data as a tare is that a nonspinning body develops a side force and yawing moment which is roll-angle-dependent.[4] I would suggest rather that the wind-tunnel balance, with spinning model mounted, serve as a flow angularity probe [viz Eq. (7)] to obtain $\{\alpha_0, \beta_0\}$. It should also be pointed out in Platou's correction equations that the use of the Magnus coefficient, C_{N_p}, with its implied linearity of force with spin rate is probably inappropriate for fin-stabilized configurations. Although bodies of revolution may evidence linearity of load with spin rate over a broad range of interest, finned bodies, in general, do not. Even if linearity were justified, the derivative, C_{Y_p}, would be required for consistency with his implied Y-axis direction.

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Reply by Author to F. J. Regan

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I WISH to thank F. J. Regan for his comments on my paper.[1] I feel that Regan agrees with my main finding that there is a normal force interaction term in the Magnus data presented in Ref. 2. The main disagreement appears to be in how to eliminate the interaction from the data. I still feel that the most accurate way to do this is to subtract the zero spin measurement from the spin measurement at the same angle of attack. This is difficult or impossible to do when the zero spin data are roll dependent. The zero spin data measurement is not impossible if it is not roll dependent.

In the case where one wishes to or is forced to correct spin data for a normal force interaction then one has the choice of my technique or Regan's technique—both have their difficulties.

In my technique one must estimate the Magnus force center of pressure or in the case of a finned body where both fin and body are rotating one must also contend with the produced Magnus couple. However, my technique does take into account the variation of average flow inclination over the body at each angle of attack.

Regan's technique eliminates the need to estimate the Magnus center of pressure, but it does assume that the flow inclination is constant in the wind-tunnel flow region traversed by the model. Since the normal force interaction in the Magnus measuring direction is very sensitive to the exact flow inclination one must be very careful in evaluating the results of this corrective technique. I would suggest that anyone evaluating wind-tunnel Magnus data where normal force interactions are suspected should attempt correction of the data using both techniques.

In closing, I would like to say that my main reason for publishing Ref. 1 was to make the reader aware that Magnus data on a self-rotating configuration can contain a normal force interaction and that a careful study of the data is necessary before one can use these data as free flight Magnus data.

The other points of disagreement are minor and need only a short comment. My sentence referred to in Regan's third paragraph should be changed to read, "The existence of a moment at zero force is indicative of a couple and in this case (Ref. 2) is due to the normal force interaction term ($N \sin \varepsilon$) acting opposite to the fin Magnus force." Also, Eq. (8) in Regan's comment is correct rather than my Eq. (2).

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Reply by Authors to A. G. Kurn

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IN the comments (see Ref. 1) on the authors' paper entitled, "Reduction of Noise from Supersonic Jet Flows,"[2] Kurn has drawn attention to his interesting experimental results on pressure fluctuations at the base of a bluff afterbody containing a sonic nozzle with the jet flow submerged in an external free-stream of a transonic wind tunnel.[3] He points out that at certain ratios of the total head of the jet flow to that of the surrounding uniform flow, a sudden reduction of discrete spectral components of the base-pressure fluctuations was observed. Based on schlieren photographs of the flow, he attributes this behavior of the base-pressure fluctuations to the modification or elimination of the periodic shedding of vortices from the bluff base of the afterbody. Since no direct noise measurements were undertaken by Kurn, the deductions about any possible changes in the radiated "far-field" noise from this flow configuration are based entirely on the corresponding behavior of the measured base-pressure fluctuations. Since the periodic vortex shedding observed by Kurn[3] and also by many others in supersonic free jet flows[4–6] has often been shown to generate discrete sound emissions, it therefore seems to be a reasonable deduction that either the disappearance or the modification in strength or periodicity of the vortex shedding in Kurn's experiments may lead to an elimination, modification, or reduction of discrete component of the related noise emission. Kurn, however, assumes similarities between his experiments and those described by the authors.[2] He then advances an alternate hypothesis that the elimination of the vortex shedding at the interface (mixing region; Fig. 7b; Ref. 2) of the inner and outer coaxial jets may be responsible for the observed noise reductions reported in Ref. 2.

The authors submit that the flow characteristics of a sonic jet exhausting into a bluff base submerged in a much larger uniform

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